Positive Fragments of Coalgebraic Logics

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Background

Modal logic

- atomic propositions
- Boolean operations
  $\vee, \wedge, \top, \bot, \neg$
- unary $\Box$
- axioms:

$$\Box(a \wedge b) = \Box a \wedge \Box b, \;\; \Box\top = \top$$

Then $\Diamond ::= \neg\Box\neg$

Positive modal logic

- atomic propositions
- lattice operations
  $\vee, \wedge, \top, \bot$
- unary $\Box$, $\Diamond$
- axioms:

$$\Box(a \wedge b) = \Box a \wedge \Box b, \Box\top = \top$$
$$\Diamond(a \vee b) = \Diamond a \vee \Diamond b, \Diamond\bot = \bot$$

($\Rightarrow$ modal operators are monotone)

$$\Box a \wedge \Diamond b \leq \Diamond(a \wedge b)$$

$$\Box(a \vee b) \leq \Diamond a \vee \Box b$$

Background

Modal logic is about Kripke frames $(X, R \subseteq X \times X)$

Equivalently, coalgebras for powerset functor $\mathcal{P}$: $\begin{cases} X \to \mathcal{P}X \\ x \mapsto \{y \in X \mid xRy\} \end{cases}$

More generally, replace $\mathcal{P}$ by any functor $T : \mathsf{Set} \to \mathsf{Set}$

Reasoning about T-coalgebras: coalgebraic modal logic

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Reasoning about T-coalgebras: coalgebraic modal logic

Goal of the talk: positive coalgebraic logic

Plan - first part

- Abstract coalgebraic logic
- Poset-enriched category theory
- Strongly finitary logic for Set-functors
- Strongly finitary logics for Poset-functors

Abstract coalgebraic logic

Context: Stone-type duality

$$\mathsf{Spa}^{op} \underset{P}{\overset{S}{\rightleftarrows}} \mathsf{Alg} \quad (\bot)$$

$$\begin{cases} \mathsf{Set}^{op} \underset{P}{\overset{S}{\rightleftarrows}} \mathsf{BA} \quad (\bot) \\ \mathsf{Poset}^{op} \underset{P'}{\overset{S'}{\rightleftarrows}} \mathsf{DL} \quad (\bot) \end{cases}$$

- P maps a set to the BA of its subsets
- S maps a BA to the set of its ultrafilters
- P' maps a poset to the DL of its upsets.
- S' associates to any DL the poset of prime filters.

Abstract coalgebraic logic

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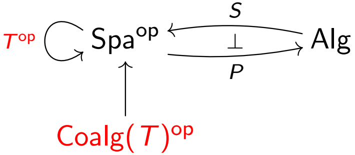

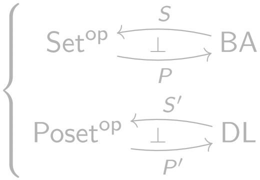

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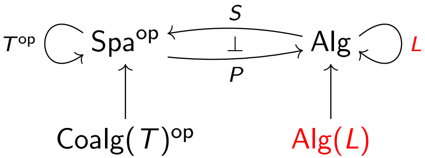

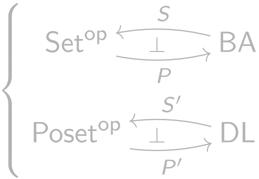

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Coalgebraic modal logic, abstractly

Syntax $L : \mathsf{Alg} \to \mathsf{Alg}$ functor

Semantics $\delta : LP \to PT^{op}$ natural transformation

Poset-enriched category theory

- Poset-enriched category: hom-sets are ordered.
  Examples: Set, BA (both with discrete order), Poset, DL (with order induced by operations)
- Poset-enriched functor: locally monotone functors (those preserving the order on the homsets).
  Example: $D : \mathsf{Set} \to \mathsf{Poset}$ the discrete functor
  Non-example: $V : \mathsf{Poset} \to \mathsf{Set}$ the forgetful functor
- Poset-natural transformation: monotone natural transformation
- Enriched adjunctions $S \dashv P$, $S' \dashv P'$
- Monadic (enriched) adjunctions

$$F \dashv U : \mathsf{BA} \to \mathsf{Set} \qquad F' \dashv U' : \mathsf{DL} \to \mathsf{Poset}$$

More on Poset-enriched category theory

Let $\mathbf{J} : \mathsf{BA}_{\mathsf{ff}} \to \mathsf{BA}$ and $\mathbf{J}' : \mathsf{DL}_{\mathsf{ff}} \to \mathsf{DL}$ be the inclusion functors of the full subcategories spanned by the algebras which are free on finite (discrete po)sets

$$\mathsf{BA}_{\mathsf{ff}} \xrightarrow{\quad \mathbf{J} \quad} \mathsf{BA}$$

Theorem

- BA *and* DL *are free cocompletions under sifted colimits of* $\mathsf{BA}_{\mathsf{ff}}$*, resp.* $\mathsf{DL}_{\mathsf{ff}}$

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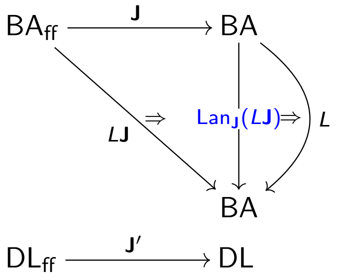

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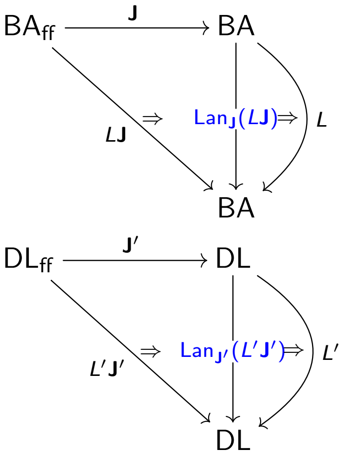

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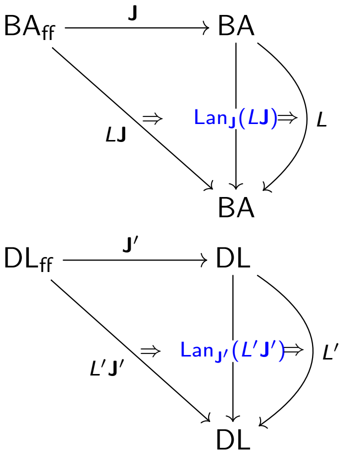

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Corollary: Both L and L' preserving sifted colimits have presentations by (monotone) operations and equations

Strongly finitary coalgebraic logic for Set-functors

Context: standard duality of propositional logic

$$T^{op} \circlearrowright \mathrm{Set}^{op} \underset{P}{\overset{S}{\underset{\perp}{\leftrightarrows}}} \mathrm{BA} \circlearrowleft L$$

- Minimal requirement: $\mathrm{Alg}(L)$ is a variety
  This happens if L itself has a presentation by operations and equations
- That is, if L preserves sifted colimits, or equivalently, if L is determined by its restriction to the finitely generated free BAs
- Define $LFn ::= PT^{\mathrm{op}}SFn$
  Then $ULFn = Set(T(2^n), 2)$ the set of n-ary predicate liftings
- Semantics $\delta : LP \to PT$ is then automatically obtained as the mate of the morphism $L \to PT^{\mathrm{op}}S$ under the adjunction $S \dashv P$
- Good properties: expressiveness, completeness, ...

Predicate liftings

Remember

Predicate liftings of arity n for a Set-functor T are natural transformations

$$\heartsuit : \mathsf{Set}(-, 2^n) \to \mathsf{Set}(T-, 2)$$

Equivalently by Yoneda lemma, elements of $\mathsf{Set}(T(2^n), 2) = UPT^{\mathsf{op}}SFn$

Definition

Predicate liftings for a Poset-functor T' (locally monotone!) of arity $\mathbb{p}$, where $\mathbb{p}$ is a finite poset, are Poset-natural transformation

$$\heartsuit : \mathsf{Poset}(-, [\mathbb{p}, \mathbb{2}]) \to \mathsf{Poset}(T'-, \mathbb{2})$$

Equivalently (by the enriched Yoneda lemma), elements of the poset $[T'([\mathbb{p}, \mathbb{2}]), \mathbb{2}] = U'P'T'^{\mathsf{op}}S'F'\mathbb{p}$

Poset-functors and their (strongly finitary) coalgebraic logic

T' : Poset $\to$ Poset **locally monotone functor**

T'-coalgebra

States: partially ordered set $\mathbb{X} = (X, \leq)$

Dynamics: monotone map $\mathbb{X} \to T'\mathbb{X}$

Logical connection: $$\mathsf{Poset}^{op} \underset{P'}{\overset{S'}{\rightleftarrows}} \mathsf{DL} \quad (\bot)$$

Logic: $(\mathbf{L}' : \mathsf{DL} \to \mathsf{DL}, \delta' : L'P' \to P'T'^{op})$

Syntax $\mathbf{L}' ::= P'T'^{op}S'\mathbf{J}'$ on free finitely generated DLs on (discrete po)sets (Dn-ary predicate liftings)
$\Leftrightarrow$ $\mathbf{L}' ::= \mathsf{Lan}_{\mathbf{J}'}(P'T'^{op}S'\mathbf{J}')$ on all DLs
$\Leftrightarrow$ $\mathbf{L}'$ preserves sifted colimits

Semantics $\delta' : \mathbf{L}'P' \to P'T'^{op}$ is the adjoint transpose of $\mathbf{L}' \to P'T'^{op}S'$ (which comes from the universal property of $\mathbf{L}'$ as left Kan extension)

Plan - second part

Two logical connections...

$$T^{op} \circlearrowright \mathsf{Set}^{op} \underset{P}{\overset{S}{\underset{\perp}{\leftrightarrows}}} \mathsf{BA} \circlearrowleft L$$

$$T'^{op} \circlearrowright \mathsf{Poset}^{op} \underset{P'}{\overset{S'}{\underset{\perp}{\leftrightarrows}}} \mathsf{DL} \circlearrowleft L'$$

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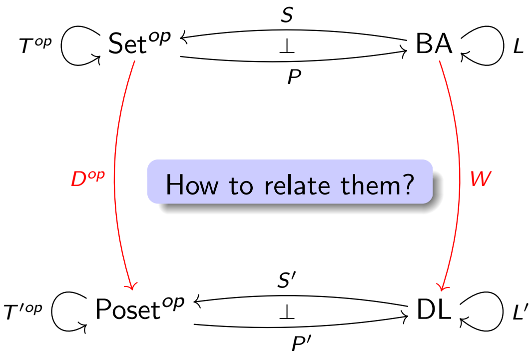

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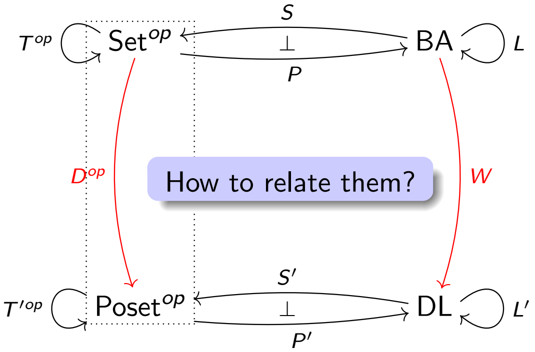

▸ Coalgebraic side

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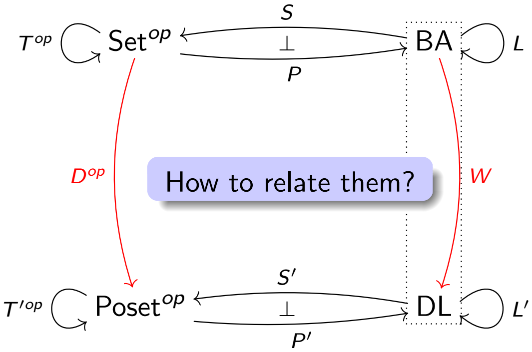

How to relate them?

▸ Coalgebraic side

▸ Logical side

The Poset-extension of a Set-functor T

Definition (B-Kurz, CALCO2011)

An *extension* of T is a *locally monotone* functor $T' : \mathsf{Poset} \to \mathsf{Poset}$ such that $DT \cong T'D$.

$$\begin{array}{ccc} \mathsf{Set} & \xrightarrow{T} & \mathsf{Set} \\ {\scriptstyle D}\downarrow & \cong & \downarrow{\scriptstyle D} \\ \mathsf{Poset} & \xrightarrow{T'} & \mathsf{Poset} \end{array}$$

An extension T' is called the *posetification* of T, if the above square exhibits T' as $\mathrm{Lan}_D DT$, the Poset-enriched left Kan extension of DT along D.

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Theorem (B-Kurz-Velebil'13)

For each Set-*functor, the posetification exists.*

Examples

Kripke functors

$$T ::= \mathrm{Id} \mid T_{X_0} \mid T_0 + T_1 \mid T_0 \times T_1 \mid T^A$$

Posetifications are as expected:

- Posetification of $\mathrm{Id}_{\mathsf{Set}}$ is $\mathrm{Id}_{\mathsf{Poset}}$
- Posetification of the constant functor at set X_0 is the constant functor at discrete poset $(X_0, =)$
- Posetification of (co)product functor is again the (co)product, this time in Poset
- Posetification of exponential functor $TX = X^A$ is again exponential in Poset

Examples (continued)

Motivating example: $T = \mathcal{P}$, the (finite) power-set functor

Posetification is the (finitely generated) convex power-set functor, with the Egli-Milner order.

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Distribution functor $\mathcal{D}X = \{d : X \to [0,1] \mid \sum_{x \in X} d(x) = 1\}$
Coalgebras: Markov chains
Posetification: $\mathcal{D}'(X, \leq)$ is $\mathcal{D}X$, with order given by

$$d \leq d' \Leftrightarrow \exists \omega \in \mathcal{D}(X \times X) \ . \ \begin{cases} \omega(x,y) > 0 \Rightarrow x \leq y \\ \sum_{y \in X} \omega(x,y) = d(x) \\ \sum_{x \in X} \omega(x,y) = d'(y) \end{cases}$$

Multiset functor $\mathcal{M}X = \{\varphi : X \to \mathbb{N} \mid \mathsf{supp}(\varphi) < \infty\}$
Coalgebras: multigraphs
Posetification: still to compute...

Intermezzo on predicate liftings

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$\heartsuit$ is *monotone* if it lifts to an enriched Poset-natural transformation.

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Proposition

Let T *be a* Set*-functor and* $T' : \mathsf{Poset} \to \mathsf{Poset}$ *an extension. Then:*

- *The set of predicate liftings of* T' *of arity* $\mathbb{p}$ *(*$\mathbb{p}$ *finite poset), is injectively mapped into the set of monotone predicate liftings of* T *of arity* $V\mathbb{p}$*.*

  In particular, the set of predicate liftings of T' *of discrete arity* Dn *embeds into the monotone predicate liftings of* T*.*
- *If* T' *is the posetification of* T*, the above mapping is a bijection.*

◂ Back to the big picture

Relating abstract coalgebraic logics

$T : \mathsf{Set} \to \mathsf{Set}$ with logic (L, δ)

Extension $T' : \mathsf{Poset} \to \mathsf{Poset}$ with $DT \xrightarrow{\alpha} T'D$ and logic (L', δ')

Definition

(L', δ') is a positive fragment of (L, δ) if there is a natural transformation $\beta : L'W \to WL$ appropriately commuting with δ and δ'

$$\begin{array}{ccccc} \mathsf{Set}^{\mathsf{op}} & \xrightarrow{P} & \mathsf{BA} & \xrightarrow{W} & \mathsf{DL} \\ {\scriptstyle T^{\mathsf{op}}}\downarrow & \swarrow\delta & {\scriptstyle L}\downarrow & \swarrow\beta & \downarrow{\scriptstyle L'} \\ \mathsf{Set}^{\mathsf{op}} & \xrightarrow[P]{} & \mathsf{BA} & \xrightarrow[W]{} & \mathsf{DL} \end{array} \quad = \quad \begin{array}{ccccc} \mathsf{Set}^{\mathsf{op}} & \xrightarrow{D^{\mathsf{op}}} & \mathsf{Poset}^{\mathsf{op}} & \xrightarrow{P'} & \mathsf{DL} \\ {\scriptstyle T^{\mathsf{op}}}\downarrow & \swarrow\alpha^{\mathsf{op}} & {\scriptstyle T'^{\mathsf{op}}}\downarrow & \swarrow\delta' & \downarrow{\scriptstyle L'} \\ \mathsf{Set}^{\mathsf{op}} & \xrightarrow[D^{\mathsf{op}}]{} & \mathsf{Poset}^{\mathsf{op}} & \xrightarrow[P']{} & \mathsf{DL} \end{array}$$

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\end{array}
$$

L' is the (maximal) positive fragment of L if β is an isomorphism.

Main result

Theorem

Consider the following:

- $T : \mathsf{Set} \to \mathsf{Set}$ a functor
- $T' : \mathsf{Poset} \to \mathsf{Poset}$ an extension of T
- (L, δ) and (L', δ') the *strongly finitary logics* of T and T'
- T' preserves *coreflexive inserters*

Then L' is the positive fragment of L.

In particular, the above holds if T preserves weak pullbacks, and T' is the posetification of T.

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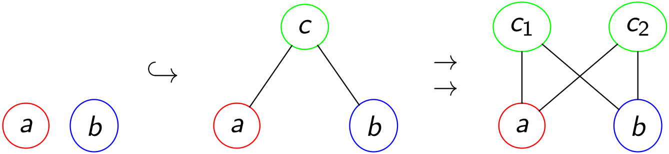
A non-example

- $T = \mathrm{Id}$ is the identity functor on Set
- $\mathbf{L} = \mathrm{Id}$ on BA
- T' is the discrete connected components functor on Poset

  It is an extension of T which does not preserve the coreflexive inserter below:



- $\mathbf{L}'$ is given by the constant functor to the distributive lattice $\mathbb{2}$
- Then $\mathbf{L}'W \to W\mathbf{L}$ fails to be an isomorphism...

Now: the (motivating) example

- $T = \mathcal{P}$ (finite) powerset functor

  Logics: LA is the BA generated by $\Box a$, for $a \in A$, wrt $\Box$ preserving finite meets.

  Semantics: $\delta_X : LPX \to P\mathcal{P}X, \quad \Box a \mapsto \{b \in \mathcal{P}X \mid b \subseteq a\}$

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- Posetification: (finitely generated) convex powerset functor

  Logics: $L'A$ is the DL generated by $\Box a$ and $\Diamond a$, for all $a \in A$, wrt $\Box$ preserving finite meets, $\Diamond$ preserving finite joins, and

  $$\Box a \wedge \Diamond b \leq \Diamond(a \wedge b) \qquad \Box(a \vee b) \leq \Diamond a \vee \Box b$$

  Semantics: $\delta'_X : L'P'X \to P'\mathcal{P}'X, \quad \begin{cases} \Box a \mapsto \{b \in \mathcal{P}X \mid b \subseteq a\} \\ \Diamond a \mapsto \{b \in \mathcal{P}X \mid b \cap a \neq \emptyset\} \end{cases}$

More examples for future study

$T = \mathcal{M}$ the (finite) multisets functor

Logic: LA is the BA generated by $\Diamond_n a$, for $a \in A$, wrt $\Diamond_n$ preserving finite joins

Semantics: $\delta_X : LPX \to P\mathcal{M}^{op}X$, $\Diamond_n a \mapsto \{\varphi \in \mathcal{M}X \mid \underset{x \in a}{\mathrm{card}}\ \varphi(x) \geq n\}$, for $n \in \mathbb{N}$

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$T = \mathcal{D}$ (finite) probability functor
Logic: LA is the BA generated by $\Diamond_q a$, for $a \in A$, wtr $\Diamond_q$ preserving finite joins
Semantics: $\delta_X : LPX \to P\mathcal{D}^{\mathrm{op}}X$, $\Diamond_q a \mapsto \{d \in \mathcal{D}X \mid \sum_{x \in a} d(x) \geq q\}$ for $q \in \mathbb{Q} \cap [0,1]$

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Semantics: $\delta_X : LPX \to P\mathcal{D}^{\mathrm{op}}X$, $\Diamond_q a \mapsto \{d \in \mathcal{D}X \mid \sum_{x \in a} d(x) \geq q\}$ for $q \in \mathbb{Q} \cap [0, 1]$

$T = \mathcal{N}$ neighbourhood functor.
Logic: LA is the BA generated by $\Box a$, for $a \in A$, no equations.
Semantics: $\delta_X : LPX \to P\mathcal{N}^{\mathrm{op}}X$, $\Box a \mapsto \{s \in \mathcal{N}X \mid a \in s\}$

Conclusions and future work

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